How Did They Calculate the Square-Root of 3 & Things Like That Before They Had Calculators, or Even Slide Rules?

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When I'm not teaching math at CWI, I am working on violins. I repair and make violins. Given my educational background (physics & math), I have an interest in the conceptual design of the violin. As far as we know, the violin was invented in northern Italy in the 1500s. Of course it had predecessors, but it appears to be unique to the area, and more importantly (for math people) to have grown out of the same design concepts that were used to design buildings constructed during the Renaissance. Think of the great Dome in Florence, Italy, as a model of what they could do without electronic calculators.

To make it even worse, they did not even have a standard measuring system; it usually varied from town to town. Nor did they have a very good way of even transmitting any information in a way we are used to. They couldn't say: "Make it 14 inches long." A 20th-century carpenter's tape measure is actually a sophisticated device, made of highly engineered materials that are resistant to changes of temperature and humidity.

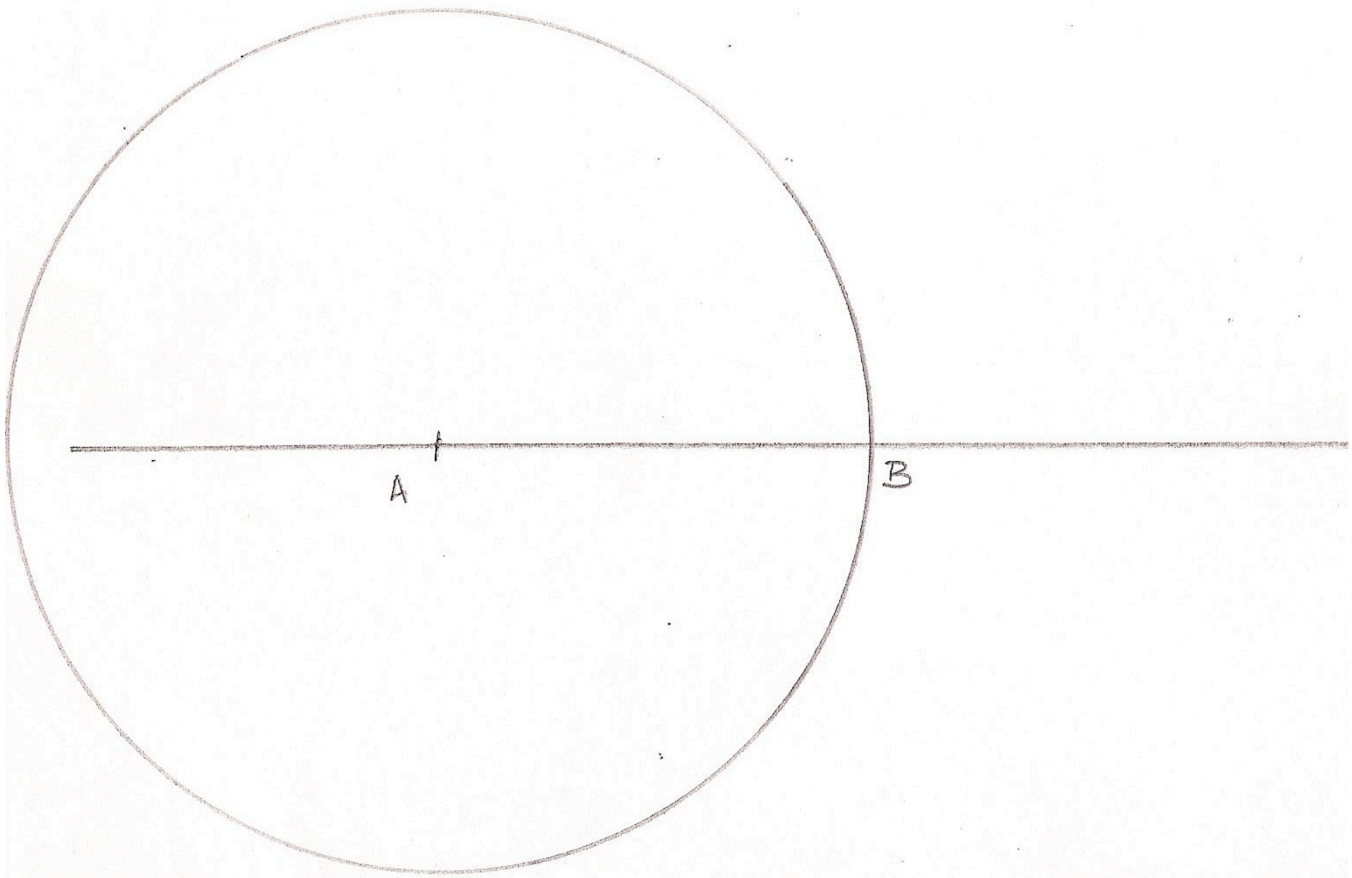
So how did they do it? They did know about Euclid and Pythagoras. They knew how to draw straight lines and circles. They thought of measurements as proportions -- ratios or fractions, in modern terms. Small-integer ratios were used to develop structure and form, such as 1/2, 3/5, and so on. Rational numbers, we call them in the math department.

Suppose, however, you were building something that required you to make a box with sides that were in the proportion of the square-root of 3 to 1. I know, that sounds weird to us, but it did come up from time to time. If you were into bad puns, you could even say it sounds irrational.

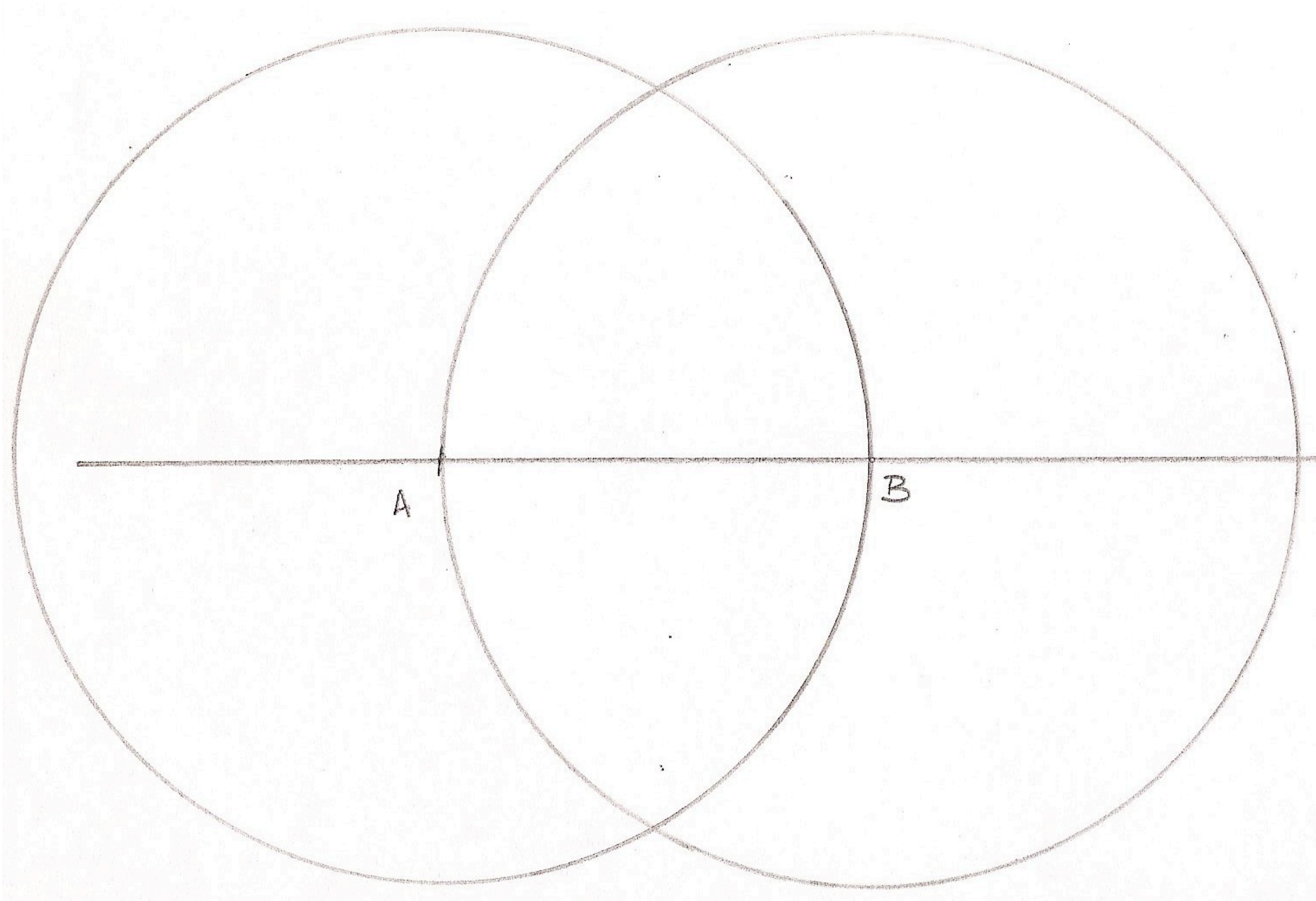
Here's one way to do it. Draw a line and put a point on it, which we'll label A here.

Now, for our purposes, let's measure over a distance of 1. One what? you might ask, and the answer would be one unit. The one length to which we want to compare our square-root-of-3 distance. This is how they would have thought of measurements. This distance compared to that distance. So, we have this --

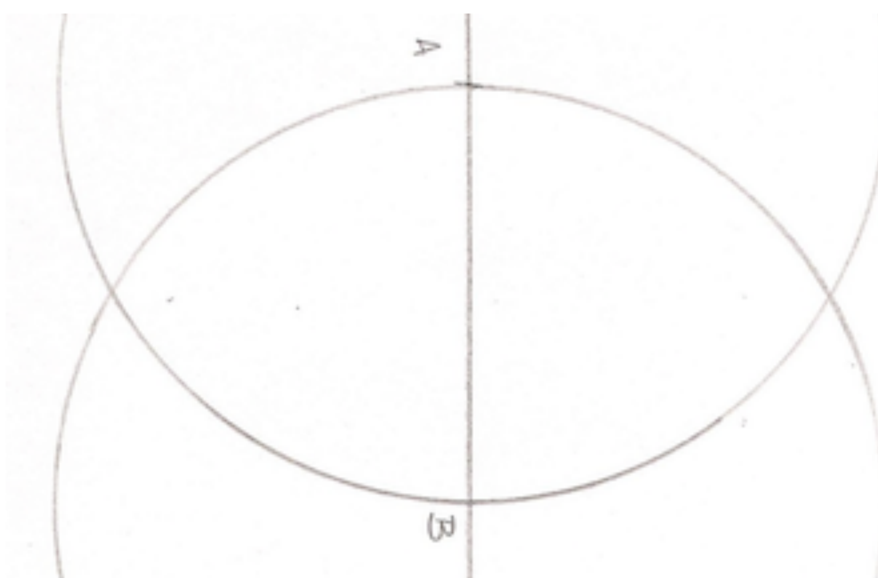
Now, with a compass centered at A, and with the pencil at B, draw a circle.



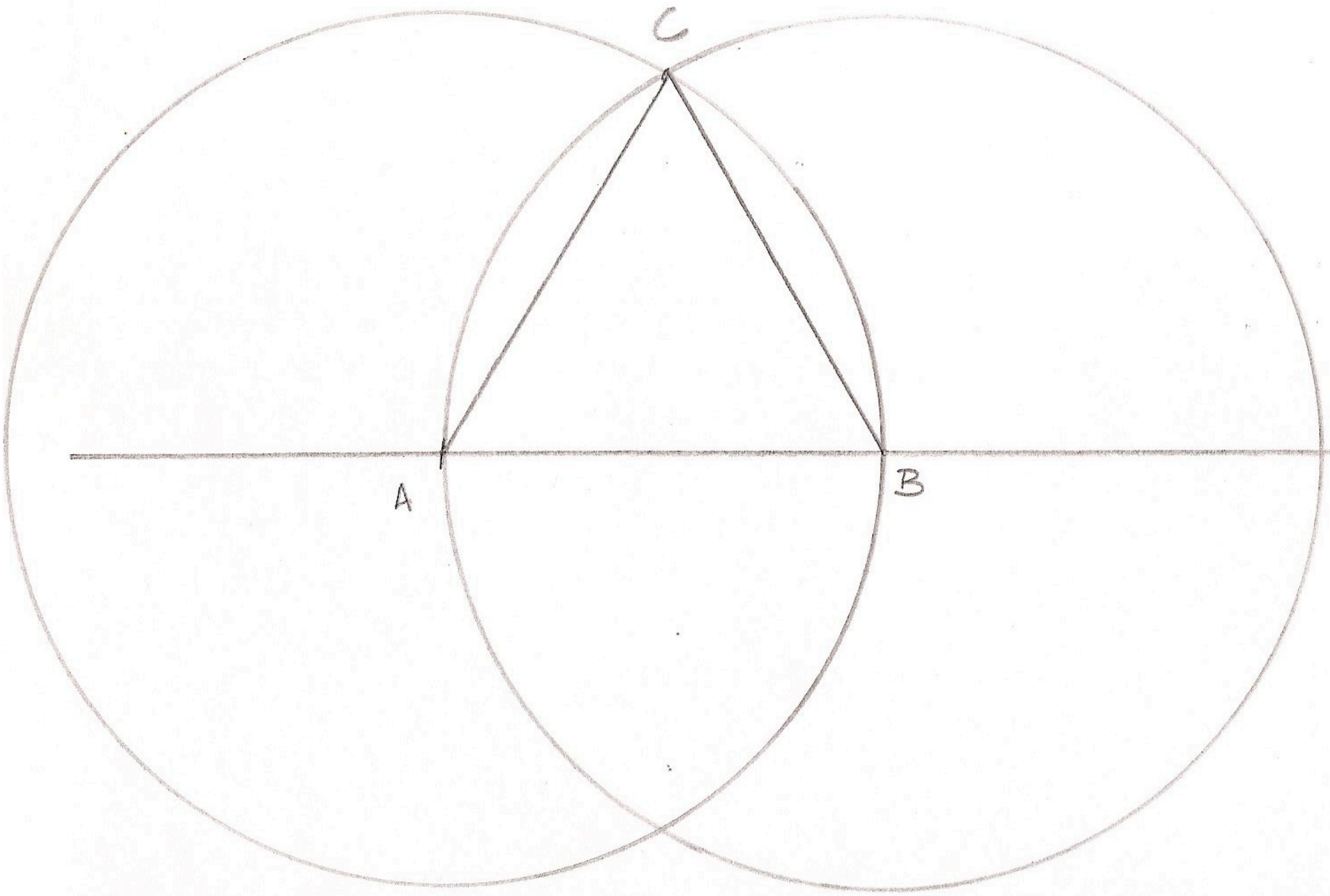
Next, put the compass center at B, the pencil at A, draw another circle.



This particular arrangement of overlapping circles is known as *vescia piscis* and had ancient religious significance later adopted by early Christians, though typically rotated 90-degrees so the overlapping-circle area was more apparently fish-shaped.

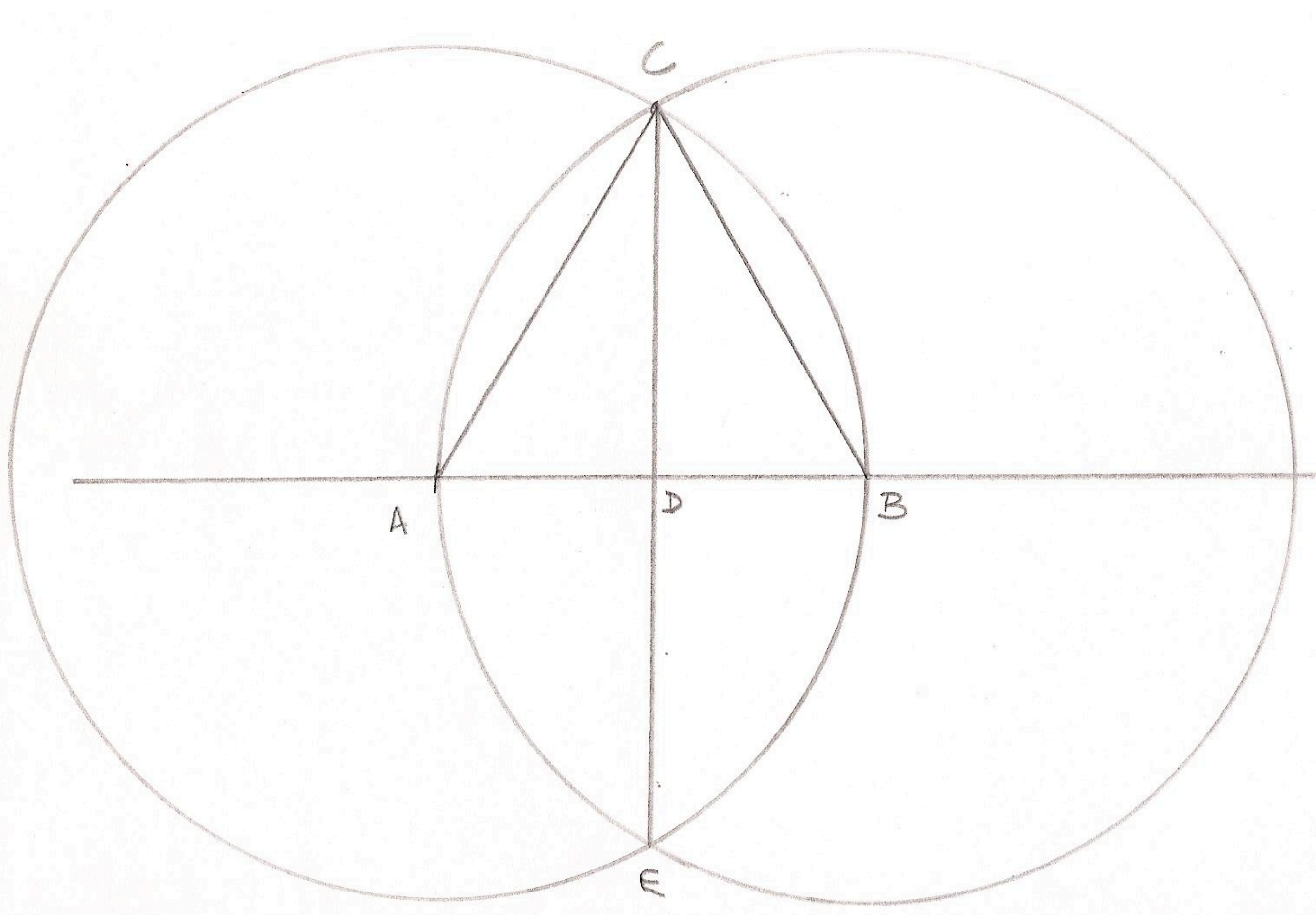


If we label the point above the line C and draw the triangle ABC, we have this --



Note that the distance from A to B is what we called 1. It is the same distance as both B to C and A to C, since they are both a radius of each individual circle. A triangle with 3 equal sides is called equilateral, and has the consequence that the internal angles are equal. Since the internal angles add to 180 degrees (or two right-angles in Euclidean-talk), each of the individual internal angles is 60 degrees.

If we draw a line from C down to the corresponding intersection point of the two circles below the horizontal line, we have this --

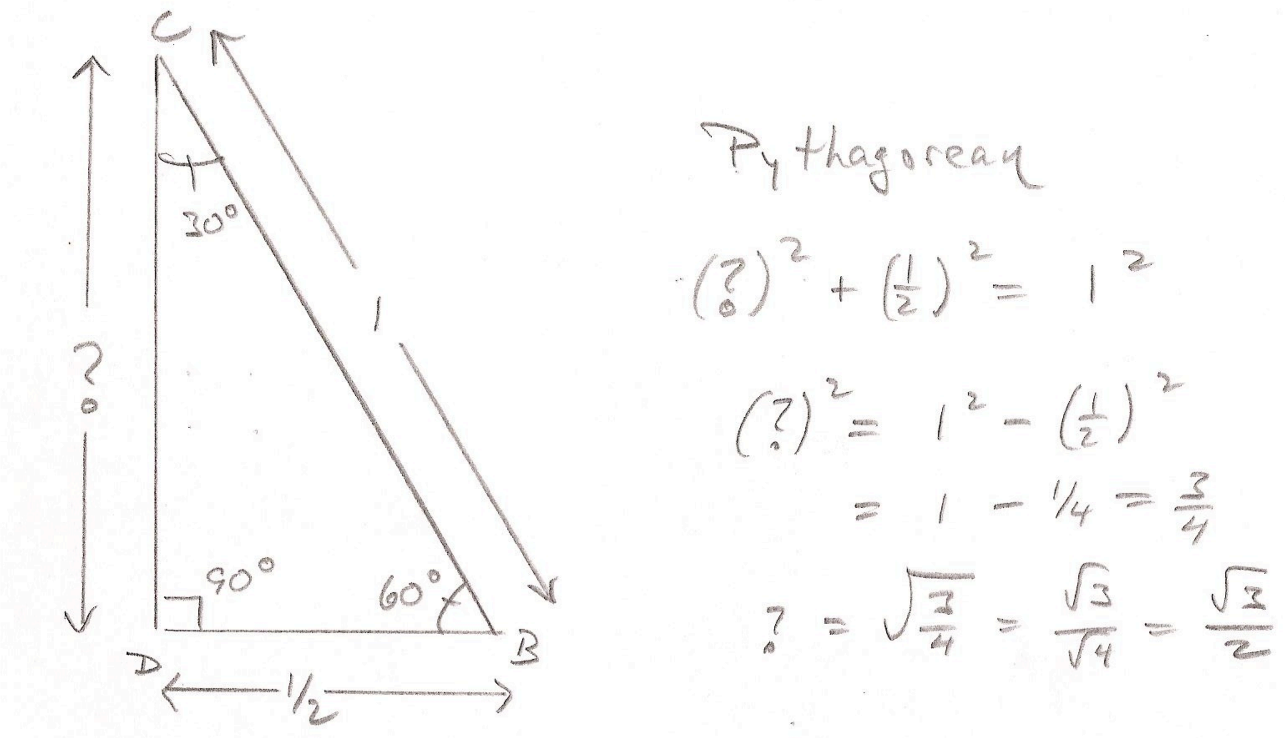


And we are done. We have our square-root of 3. Let's examine it.

When we drew the line CE we were following the Euclidean method for creating a perpendicular bisector. CE evenly divides AB at a right angle. D is the mid-point of AB.

Since we have split the equilateral triangle, we have two equal triangles ADC and BDC. Lets look at triangle BDC. The angle at B is still 60 degrees, the angle at D is 90 degrees (being perpendicular), and the angle at C is 30 degrees, being half of 60. This is the famous 30-60-90 triangle you can buy in plastic at the bookstore or through drafting supply stores.

Let's consider the sides of BDC. The side BC is still 1. The side DB is 1/2, because D as at the midpoint of AB which is of length 1. What about the length DC? We can use the Pythagorean Theorem to determine that it is half of the square-root of 3. See the illustration for the calculation.



Since we have used circles, we can see by symmetry that the length DE is equal to the length DC, or also one-half of the square-root of 3.

So the distance CE is one-half of the square-root of 3 plus one-half of the square-root of 3, or the square-root of 3.

Therefore, in proportions, CE to AB is the square-root-of-3 to 1. And we have constructed our desired length of the square-root of 3. These can be drawn to any scale, and the sizes can be transferred to the work with dividers (a compass).

And if you simply wanted the root-3 to 1 ratio, you don't need to draw the complete circles, just the two overlapping arcs, and draw the lines.

Pretty quick to do. No batteries required.